Physics 303 Classical Electrodynamics
Practice questions and problems set 2

1. **Gauge Invariance** A static electric field is defined by a simple scalar potential. Show that it is Gauge invariant if I add a constant to the potential. A Magnetic field is defined by the curl of a vector potential $\vec{A}$ Show that it is Gauge invariant if a vector field derived from a gradient is added to it. Prove this for a 4-potential .

2. **Invariants** Write down the electromagnetic field tensor $F_{\mu\nu}$ explicitly as a matrix. Find the invariants of this field (i.e., find its Eigen-Values) and show they are $\mathbf{E^2} - \mathbf{B^2}$ and $\mathbf{E}.\mathbf{B}$

3. Apply the method of images for a charge q at a distance d from the surface of a metal of radius a. Find the surface charge density on the sphere along with the position and value of the image charge q'

4. Consider a vector $\mathbf{A} = \hat{r}r^n$ where $\hat{r} = \mathbf{r}/r = \hat{i}x + \hat{j}y + \hat{k}z$. Find $\nabla.\mathbf{A}$. (i) What is the value of the divergence for $n = 1$ in a space of 3 dimensions and 2 dimensions . (ii) What is the value of the divergence for $n = -2$ and $n = -3$

5. A parallel plate capacitor of area $A = l \times w$ with separation d is filled partially with a dielectric slab of dielectric constnat e_1 find out what happens when the condenser plates are set to a constant voltage.

6. **Charged Ring** Consider a charged ring with total charge q located at r_0, θ_0 from the origin. Evaluate the potential on the Z - axis above the ring . Also evaluate the potential at any point r, θ. Hint: When we did multi-pole expansion we expanded $\frac{1}{|\vec{r}-\vec{r'}|}$ as a series. The series was later identified with Legendre polynomials for two regimes $r > r'$ and $r < r'$. Use this to get $V(r, \theta)$. Along the z-axis you can use a more elementary technique to directly evaluate the field.

7. Write down the Lorentz and Coulomb Gauge fixing conditions. In these cases what is the form of wave equation you expect for the potentials.

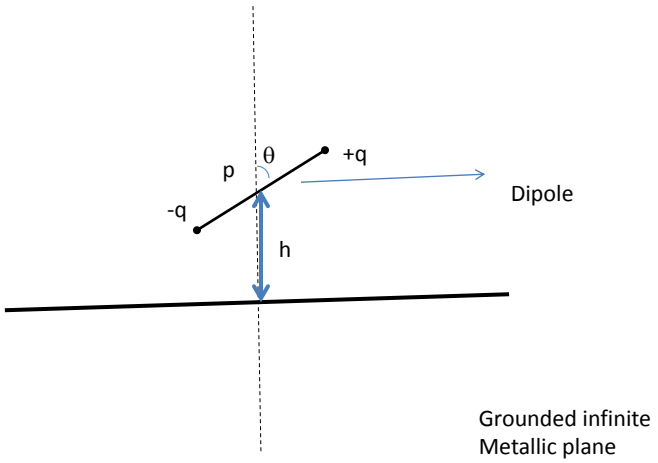

Figure 1: A dipole above a grounded infinite metallic plane.

8. Consider a dipole above a metallic plane . (a)Sketch the image charges that will uniquely help you establish the potential above the metal plane. (b) What is the field due to a dipole at an arbitrary position vector $\vec{r}$ assuming the center of the dipole is at the origin. (c) Write down the interaction energy between two dipoles of moment p1 and p2. (d) What is the work done to assemble the dipole

in the configuration given above. You can write the answer directly if you can justify the answer with appropriate physical reasons.

9. A charge Q is attached to a mass spring system that oscillates with frequency $\sqrt{\frac{k}{m}}$. If the oscillator is oscillating use the Larmor formula to find the power of the emitted radiation.

10. Assume a rectangular sheet of electrons with a sheet density n forming a 2-D metal. An electric current is driven along its length parallel to the x-axis. A magnetic field is applied in the $\hat{z}$-direction. Find the voltage called the Hall voltage in terms of the external field $\mathbf{B}$ electronic charge e and the carrier density n.
    Now assume $\gamma \sim 1$ but at the same time v the speed of electrons in the system above are not too small to ignore relativistic effects. By moving to a frame where thew current is zero show that the Hall effect stems from relativistic transformation of fields.

11. Review Feynman's thought experiment described in the class. Discuss the possible outcome of the experiment in terms of EM field concepts you already know.

    Look at problems in Schwartz.