Physics 102 Electromagnetism

Practice questions and problems Tutorial 1 ad 2

1. Consider a vector field $\vec{F} = (2xz^3 + 6y)\hat{\imath}) + ()6x - 2yz)\hat{\jmath} + (3x^2z^2 - y^2)\hat{k}$. Prove this is a conservative field.

   Solution: prove the curl is zero.

2. Feynman makes a remark that matter is usually neutral. If someone creates around $\sim$ 1% disturbance of a charge imbalance in a human being the electrostatic forces are big enough to lift planet earth. Make a crude calculation to prove or disprove this statement. You will have to setup the problem qualitatively e.g, assume a person weights 60Kg and is composed of 90% water and estimate the force assuming an equal charge is present in earth. You need to only worry about the order of magnitude and not the exact values. (Such problems are called Fermi problems. Find out what are Fermi problems)

   Solution: Water has 2 hydrogen atoms and one oxygen weighing 18 atomic units. 1 mole of water is 18 gms and has $\sim 6 \times 10^{23}$ molecules. Assume 1% of water weighing 60Kg is charged and make an estimate.

3. Consider a vector field $\vec{C}$ derived from a curl of another vector $\vec{B}$. Find the divergence of this vector.

4. Consider a vector $\mathbf{A} = \hat{r}r^n$ where $\hat{r} = \mathbf{r}/r = \hat{i}x + \hat{j}y + \hat{k}z$. Find $\nabla.\mathbf{A}$. (i) What is the value of the divergence for $n = 1$ in a space of 3 dimensions

and 2 dimensions . (ii) What is the value of the divergence for $n = -2$ and $n = -3$

Solution:

$$\mathbf{A} = \hat{r}r^n = \hat{\imath}xr^{n-1} + \hat{\jmath}yr^{n-1} + \hat{k}zr^{n-1}$$
$$\frac{\partial A_x}{\partial x} = r^{n-1} + (n-1)xr^{n-2}\frac{\partial r}{\partial x}$$
$$\frac{\partial r}{\partial x} = \frac{\partial\sqrt{x^2+y^2+z^2}}{\partial x} = 2x/2r = x/r$$
$$\frac{\partial A_x}{\partial x} = r^{n-1} + (n-1)xr^{n-2}x^2$$

Hence divergence of A is

$$\nabla.\mathbf{A} = 3r^{n-1} + (n-1)r^{n-3}(x^2+y^2+z^2) = (2+n)r^{n-1}$$

. (i) 3, 2, 1

(ii) 0, $-1/r^4$

5. Two long thin wires carry a linear has a charge density λ_1, λ_2 per unit length. Find the force per unit length due to wire 1 on wire 2 if they are separated by a distance b

Solution:

Draw a cylinder as a Gaussian Surface. The flux on the surface of the cylinder is

$$E 2\pi r L = \frac{\lambda_1 L}{\epsilon_0}$$

Hence

$$E = \frac{\lambda_1}{2\pi\epsilon_0 r}$$

The charge per unit length of wire 2 is λ_2 hence

$$(F/L) = \frac{\lambda_2 \lambda_1}{2\pi\epsilon_0 b}$$

6. i) What is the energy stored in a capacitor with capacitance C charged to a value Q ? ii)A parallel plate capacitor of area $A = l \times w$ with separation d. Evaluate the capacitance and charge at a fixed voltage when the gap if filled with (i) air and (ii)with a dielectric FR-4 (the light green plastic in PCBs) of dielectric constant $\epsilon = 4$.

Solution:

Work done in charging

$$dw = v dq$$

but

$$V = Q/C$$

$$\int dw = \int_0^Q Qdq = Q^2/2C$$

.

Work out the converse when Q is fixed V is varied and prove $U = (1/2)CV^2$.

As an exercise derive the energy stored in terms of electric field betwen the parallel plates of a capacitor

7. A point q charge is placed at a distance d before a grounded metal plane. (a)Use the image method to write the potential (b)solve for the charge density and the force on the charge. (c). Suppose there are 2 grounded planes separated by a distance d and you can place a charge anywhere in-between. Find the force on the charge.

Solution: Suppose the charge is above distance d above an XY plane at $Z = 0$ then there is an image charge $-q$ at $-d$. Hence the potential is

(a)

$$V(x, y, z) = \frac{1}{4\pi\epsilon_0}[\frac{q}{\sqrt{x^2 + y^2 + (z-d)^2}} - \frac{q}{\sqrt{x^2 + y^2 + (z+d)^2}}]$$

This is obvious if you look at the potential for a dipole which is zero at the mid point.

(b) We showed in class for a metal all charge density is at the surface with fields normal to the surface. We know

$$\vec{E_n} = \frac{\sigma}{\epsilon_0}\hat{n}$$

$$|\vec{E_n}| = -\left[\frac{\partial V}{\partial n}\right]_{surface}$$

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n}$$

In this case the surface is at $Z = 0$

$$\sigma = -\epsilon_0 \left[\frac{\partial V}{\partial z}\right]_{Z=0}$$

$$\frac{\partial V}{\partial z} = \frac{1}{4\pi\epsilon_0}[\frac{-q(z-d)}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{q(z+d)}{\sqrt{x^2+y^2+(z+d)^2}}]$$

at $Z = 0$

$$\sigma = \frac{-qd}{2\pi(x^2+y^2+d^2)^{3/2}}$$

The force is simply due to the image charge

$$\vec{F} = \frac{-1}{4\pi\epsilon_0}\frac{q^2}{(2d)^2}\hat{k}$$

8. A parallel plate capacitor of area $A = l \times w$ with separation d is filled

partially with a dielectric slab of dielectric constant e_1 find out what happens when the condenser plates are set to a constant charge Q. (Hint : use the result for energy stored in terms of charge and capacitance in a previous problem)

Solution:

$$C_0 = \frac{\epsilon_0 l w}{d}$$

When the dielectric is inserted to a length say x

$$C = \frac{\epsilon_1 \epsilon_0 x w}{d} + \frac{\epsilon_0 (l-x) w}{d}$$

The potential stored

$$U = 1/2CV^2 = (1/2)\left(\frac{\epsilon_1 \epsilon_0 x w}{d} + \frac{\epsilon_0 (l-x) w}{d}\right) V^2$$

There is a force due to gradient of potential across the capacitor.

$$F = -gradU = -(1/2)\left(\frac{\epsilon_1 \epsilon_0 w}{d} + \frac{\epsilon_0 - w}{d}\right) V^2$$

$$F = -(1/2)V^2 \frac{\epsilon_0 w(\epsilon_1 - 1)}{d}$$

This solution is deliberately given at constant voltage. Try doing at constant charge as posed in the problem.

9. Consider a circular sheet of charge with density σ per unit area. If the radius of the disc is $b$find the field at a distance a from the circle. Evaluate the field at point a when the sheet is of infinite dimensions.

Solution: Area of a circle $A = \pi r^2$ implies an area element $dA = 2\pi r dr$ Consider a small annular region of a circle of mean radius r. The electric field due to this at an axial point a will be

$$dE = \frac{2\sigma\pi r dr cos\theta}{4\pi\epsilon_0 l^2}$$

$$tan\theta = r/a \implies dr = asec^2\theta d\theta$$

$$cos\theta = a/l \implies l = asec\theta$$

Substituting these we have

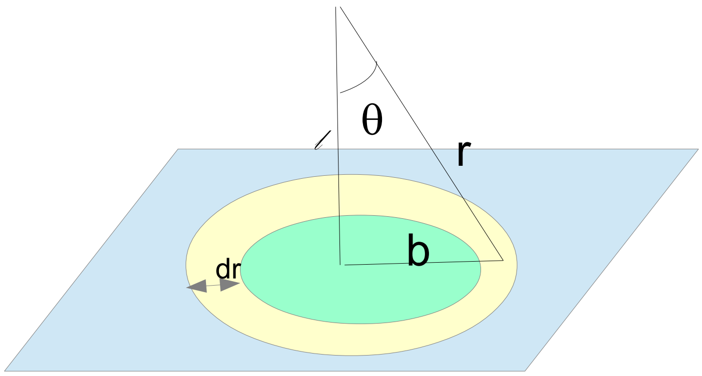
$$|d\vec{E}_\perp| = \frac{\sigma\, atan\theta\, asec^2\theta\, d\theta}{2\epsilon_0 a^2 sec^2\theta}$$

$$|d\vec{E}_\perp| = \frac{\sigma\, sin\theta\, d\theta}{2\epsilon_0}$$

The total field is integrated from $0 \to b$

$$|\vec{E_\perp}| = \int_0^b \frac{sin\theta\, d\theta}{2\epsilon_0}$$

the value of θ in this range is $0 \to \pi/2$



Blue region indicates charge
White annulus is a region
Considere for integration

$$|\vec{E_\perp}| = \int_0^{\pi/2} \frac{sin\theta\, d\theta}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0}\left[-cos\theta\right]_0^{\pi/2} = \frac{\sigma}{2\epsilon_0}$$

Find out the solution for finite sheet. In the limit it is infinite find the field.

Some of you did this with an algebraic notation and others solved the potential. Any one who gives alternate solutions in Latex will get it posted on the course web page with their name on it. LaTeX is a command based editor derived from TeX. As an example a command

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$\vec{\nabla}\times \vec{A} = \vec{B}$
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where anything between the two $ signs means it is a mathematical expression and the commands will produce approriate equations.

The above command when compiled will produce $\vec{\nabla}\times\vec{A} = \vec{B}$ **You can download a free book like Not So short introducution to Latex. The software is also free.**

10. A large parallel plate capacitor has a separation d and is lying in the X-Y plane with the top plate positive and the bottom plate negative. A ball of mass m is charged to a value $+Q$ and released from the top plate with a speed v along the x-axis just below the top plate. Solve for the trajectory of the ball if $\vec{g}$ is acting downward along the z-axis.

Solution: Solve this using kinematic equations like projectile motion except there is a g and electric field.

11. Arrive at Poisson and Laplace equations from the differential form of Gauss' law.

12. When charges accelerate they radiate energy. This means electrons moving around the nuclei like a planetary motion will not be stable. If one considers only a static arrangement of an electron and a proton it will not be stable. The text showed some arguments using Gauss' law. Can you assume Laplace's equation holds in the region between the two charges and show a condition for stable equilibrium is not satisfied.

13. A dipole of moment $\vec{p}$ is subject to a uniform external field. Find the force and torque on the dipole. What is the potential energy of a dipole in external field.

14. You are standing on a very long highway and watching cars that approach you. When the cars are too far away the head lights appear to be a single lamp. When they approach a little closer you are able to see the two headlights separately. Can you find out if you can use this to comment on dipole fields.